Numerical Flow Simulation of Viscoplastic Slurries and Design Criteria for a Tape Casting Unit

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Numerical simulations have been carried out using the finite element method (FEM) for the forming flow of ceramic tapes. The flow domain encompasses both the slurry reservoir and the doctor-blade region with free surface and is fully two-dimensional. The material of this study is an organic-bonded alumina slurry used in a previous experimental investigation and is modeled as a viscoplastic Bingham fluid with a yield stress. For different substrate speeds, the entire flow domain is analyzed and the extent and shape of yielded/unyielded regions are found. The computed free surface profiles are in close agreement with the experimental ones. Large vortices appear in the reservoir and their intensity is computed under different operating conditions. The lubrication approximation theory (LAT) is then used to deduce a new reservoir design with tapered walls avoiding recirculation. Subsequent FEM studies show the adequacy of the new design to achieve certain criteria such as elimination of vortices and an evenly distributed unyielded region. It is proposed that a combination of LAT and FEM techniques be used for design of ceramic tape casting equipment.

I. Introduction

A MULTITUDE of ceramic products for the electronics industry such as microcapacitors, covers for integrated circuits, diaphragms, etc., are produced by the tape casting technique. An extensive description of the process is given by Williams. The forming process of ceramic layers is an essential step which influences the properties of the semifinished articles (especially the dried ceramic sheets) as well as the final products.

The production of ceramic sheets can be carried out as shown in Fig. 1 with one of the following processes: (a) doctor-blade casting, (b) batch casting, (c) rotation casting. The last is a newly developed process for continuous production on a solid glass sheet and is especially suited for making thin multilayer tapes. Each process comprises (I) the forming of a liquid ceramic sheet by using refined ceramic slurry on a support belt or a glass sheet, (II) the drying of the wet sheet, and (III) its removal as a dry sheet, as shown schematically in Fig. 1. Usually the forming units are based on the blade-coating principle (see Fig. 2).

Detailed information of the forming process is especially of interest for the production of grain-oriented ceramics and for slurries with tendencies to build textures. In both cases the particle orientation degree should be the same all over the ceramic tape either to achieve certain properties (e.g., high density) or to avoid failures (e.g., tension tears). Particle alignment in the green sheet is caused by the forming process as shown by Watanabe et al. The degree of particle orientation showed dependence on the shear rate (main flow parameter influencing the orientation) under a certain value of the shear rate. Seidemann

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described the correlation between the shear rate distribution of a viscoplastic slurry and the degree of orientation, which shows no particle orientation inside the plug region and increasing orientation degree with increasing shear rate. Consequently, regions of small shear rates, such as, for example, plug regions in viscoplastic flow, have to be avoided to reach an equal degree of particle orientation.

Mathematical modeling and numerical simulations are the favored investigation methods because of the small size of the area of interest, which makes it very hard for detailed measurements. Usually, the support belt speed and liquid thickness can be measured for a certain design. Numerical simulation can provide other critical details, such as regions of recirculation, yielded/unyielded regions in cases of viscoplastic materials, and stresses in the flow field.

Previous theoretical analyses have used only one-dimensional models and simple rheological equations (Newtonian, Bingham, or power-law fluids) to describe and design the blade channel. However, the complete flow field including the reservoir and the meniscus at exit is hardly one-dimensional, and rheological data for the slurries used are seldom fitted by such simple equations.

The assumption of ceramic slurries behaving as viscoplastic fluids is of particular interest, because such behavior can be approximated by the Bingham model with a yield stress \( \tau_y \) and a plastic viscosity \( \mu \). This model assumes that a fluid behaves as a solid body at stresses below \( \tau_y \).

In recent years, a renewed interest has surfaced for the flow analysis of viscoplastic materials. Following a seminal paper by Bird et al., who give a thorough review of viscoplastic materials up to the early 1980s, a series of investigations appeared to study viscoplastic flow, usually with the Bingham model (see recent review by Abdali et al.). A novel constitutive equation has been proposed by Papanastasiou, equally valid in both yielded and unyielded regions, thus making it easier to track down such surfaces. Such a model has also been used by Abdali et al. to study the entry and exit flows of Bingham plasitcs in planar and axisymmetric contractions.

Whether or not a true yield stress exists at low shear rates has been the subject of intense debate in the scientific community. Earlier investigations of alumina slurries showed that no yield stress could be found. These slurries exhibited a sudden increase in the viscosity at low shear rates (below 1 s\(^{-1}\)), which might be interpreted as corresponding to a yield stress. Also the difficulty of making such measurements in the low shear region adds another complication. Special equipment is needed and avoidance of large errors in the data is not guaranteed. Usually reliable data are available only at higher shear rates, and yield stresses are assumed and extrapolated. This seems to be a reasonable engineering approximation, and the assumption of no deformation under a certain yield stress value in rheological equations leads to acceptable results for the description of viscoplastic materials such as ceramic slurries.

The assumption of viscoplastic behavior with a yield stress for ceramic slurries can thus be viewed as a worst-case scenario, taking into account that the presence of yield surfaces may be associated with lack of particle orientation and the production of ceramic tapes with uneven particle distribution and alignment.

It is the purpose, therefore, of the present work to perform simulations on the process of ceramic tape casting for a particular alumina slurry used in an earlier experimental work and assumed to behave as a viscoplastic fluid with a yield stress. The emphasis will be on analyzing the entry flow from the reservoir into the doctor-blade region as well as the exit region where a free surface (meniscus) forms. The analysis will be used to deduce new designs able to meet certain criteria deemed optimal for better operation.

II. Experimental Data

Experiments were conducted on a tape casting machine (see Fig. 1(a)). The forming unit uses a blade casting tool (Fig. 2). The channel gap \( H \) was 1.5 mm and the channel width in the third dimension was 200 mm. A support belt was used having a width of 240 mm. Its speed was kept constant at \( V_B = 7.58 \) mm/s by electronic control.

The slurry used was an organic-bonded alumina slurry. It consists of \( \alpha\)-Al\(_2\)O\(_3\) powder, cellulose acetate ( binder), dibutyl phthalate (plasticizer), fish oil, acetone (solvent), and cyclohexanone (solvent). The density of the slurry was measured at \( \rho = 1825.5 \pm 0.7 \) kg/m\(^3\). The slurry rheology was determined with a Couette rheometer (Rheotest III) employing a system of coaxial cylinders. The measured data were assumed to obey the Bingham model

\[
\tau = \tau_y + \mu \gamma
\]

with a yield stress (found by extrapolation) \( \tau_y = 12.1 \) Pa and a viscosity \( \mu = 4.46 \) Pa\(\cdot\)s in the range of shear rates \( 0 < \gamma < 10 \) s\(^{-1}\). Surface tension measurements on a ring tensiometer gave a value for the surface tension of the slurry \( \sigma = (29.3 \pm 0.15) \times 10^{-3} \) N/m.

All measurements and experiments were undertaken at 20°C room temperature. More details about the experiments, the refinement process, and further data about the slurry (e.g., particle analysis) are given elsewhere.

III. Mathematical Modeling of the Forming Process

(1) Conservation and Constitutive Equations

The analysis of ceramic tape casting will be carried out under the assumptions of steady-state, two-dimensional, laminar, and isothermal flow with free boundaries. The conservation equations of mass and momentum for an incompressible fluid are

\[
\nabla \cdot \mathbf{v} = 0
\]

\[
\rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nabla \cdot \tau + \rho \mathbf{g} \tag{3}
\]

where \( \mathbf{v} \) is the velocity vector, \( p \) is the pressure, \( \tau \) is the extra stress tensor, \( \rho \) is the density, and \( \mathbf{g} \) is the acceleration due to gravity.

The constitutive equation that relates \( \tau \) to the velocity gradient \( \nabla \mathbf{v} \) is a modified Bingham viscoplastic equation proposed by Papanastasiou incorporating an exponential term to control the stress growth before yielding and is written as

\[
\tau = \mu + \frac{\tau_y}{|\gamma|} \left(1 - \exp(-m|\gamma|)\right)\gamma
\]

where \( \mu \) is a constant viscosity, \( \tau_y \) is the yield stress, and \( m \) is the stress growth exponent. The magnitude \( |\gamma| \) of the rate-of-strain tensor \( \gamma = \nabla \mathbf{v} + (\nabla \mathbf{v})^T \) is given by

\[
|\gamma| = \sqrt{\frac{1}{2} \mathbf{H}^T \mathbf{H}} = \left(\frac{1}{2} \mathbf{H}^T \mathbf{H}\right)^{1/2}
\]
where II is the second invariant of $\bar{\tau}$. Equation (4) approximates the von Mises criterion of a plastic solid for relatively big exponent $m (m \geq 100)$, and holds uniformly in yielded and unyielded regions. It has no discontinuities in the apparent viscosity function and shows good convergence computationally. The one-dimensional analogue of Eq. (4) in simple shear flow gives

$$\tau = \mu \dot{\gamma} + \tau_c (1 - e^{-m\dot{\gamma}})$$

(6)

where $\tau$ and $\dot{\gamma}$ are the shear stress and shear rate, respectively. Figure 3 shows a graphical representation of Eq. (6) for different values of the exponent $m$. Clearly, for $m = 100$ the above equation mimics the ideal Bingham plastic. In subsequent analyses, we use $m = 200$, since previous studies have shown that the results are independent of $m$ for $m \geq 100$.

To track down yielded/unyielded regions, we shall employ the criterion that the material flows (yields) only when the magnitude of the extra stress tensor $|\bar{\tau}|$ exceeds the yield stress $\tau_c$, i.e.

Yielded

$$|\bar{\tau}| > \tau_c$$

(7a)

Unyielded

$$|\bar{\tau}| \leq \tau_c$$

(7b)

where II is the second invariant of the extra stress tensor. Note that previous work by Papanastasiou and Beverly and Tanner employed the criterion of the second invariant of the rate-of-strain tensor $\dot{\gamma}$ exceeding an arbitrarily small value $\dot{\gamma}_c$ close to zero. As argued by Abdall et al. such a choice may lead to noncomparable and/or ambivalent results. On the other hand, using as a criterion the value of yield stress, which is not arbitrary but a given constant, reduces the uncertainty of using values very close to zero, which may be numerical noise.

(2) Dimensionless Groups

Before proceeding with the boundary conditions, it is interesting to examine the relevant dimensionless numbers in the process. These are useful for comparisons with other equipment and also for scale-up purposes. The dimensionless groups are calculated by taking as a characteristic length the coating height $h$ and as a characteristic velocity the support belt speed $V_b$ (note that $V_b h$ corresponds to the volumetric flow rate $Q$ per unit width $W$).

The relative importance of inertia forces compared to viscous forces in the equation of momentum is assessed by the Reynolds number, defined by

$$Re = \frac{\rho V_b h}{\mu}$$

(8)

For the viscoplastic materials used in this work, $Re << 1$, and the creeping flow approximation is valid. Therefore, inertia terms are not necessary but nevertheless are included in the momentum equation for completeness.

For materials with yield stress obeying the Bingham model, it is appropriate to introduce a dimensionless Bingham number as suggested by Bird et al., in equivalence to the Reynolds number, i.e.

$$Bi = \frac{\tau_c h}{\mu V_b}$$

(9)

Note that for purely viscous fluids, $\tau_c = Bi = 0$. However, at the other extreme of an unyielded solid, $Bi \rightarrow \infty$.

The relative importance of gravity forces compared to viscous forces in the equation of momentum is assessed by the Stokes number, defined by

$$St = \frac{\rho g h^2}{\mu V_b^2}$$

(10)

For the viscoplastic materials used in this work, $St = O(1)$, and the gravity term has to be included in the equations.

The relative importance of pressure forces compared to viscous forces in the equation of momentum is assessed by the Hagen number, defined by

$$Ha = \frac{\frac{dP}{dx} h^2}{\mu V_b^2}$$

(11)

For viscoplastic drag flows, $Ha = 0$. However, a combined pressure and drag flow with $Ha \neq 0$ may result in different yielded/unyielded regions as shown in the Appendix.

The relative importance of surface tension effects compared with viscous forces on the free boundary is assessed by the capillary number, defined by

$$Ca = \frac{\mu V_b}{\sigma}$$

(12)

where $\sigma$ is the surface tension. Values of $Ca \rightarrow 0$ show a free surface flow dominated by surface tension effects; on the other extreme of $Ca \rightarrow \infty$, viscous forces are dominant and surface tension can be safely neglected. For the viscoplastic materials used in this work, $Ca = O(1)$, and surface tension has to be included for the determination of the free surface.

(3) Boundary Conditions

The solution of the conservation Eqs. (2) and (3) and constitutive equation (4) is possible only after a set of boundary conditions has been imposed on the flow domain. With regard to Fig. 4, the boundary conditions for flow analysis are the following: (1) along the support belt AB, no slip velocity, i.e., $v_s = V_b$, $v_r = 0$; (2) along the exit BC, zero surface tractions imposed and $v_r = 0$; (3) along the free surface CD, a kinematic boundary condition of no-cross flow is imposed, i.e.

$$\vec{n} \cdot \vec{v} = 0$$

(13)

where $\vec{n}$ is the unit outward normal vector to the surface. Also, a dynamic boundary condition for surface tension effects is necessary, i.e.
Fig. 4. Schematic diagram of the tape casting equipment along with notation and boundary conditions for flow analysis.

\[-p \vec{n} \cdot \vec{v} + \frac{1}{\text{Ca}} \frac{d}{ds} \frac{d}{ds} \rho = 0\]  

(14)

where \( p \) is the ambient pressure and \( \vec{v} \) is the unit tangent vector at the surface that varies with distance \( s \) along the surface and points in the direction of increasing \( s \). In Eq. (14) the pressure and stresses have been made dimensionless by dividing them by the viscous stress \( \mu V_0 h / \text{Ca} \): (4) along the channel (blade) wall DE, no slip velocity, i.e., \( v_x = v_y = 0 \); (5) along the reservoir right wall EF, no slip velocity, i.e., \( v_x = v_y = 0 \); (6) along the reservoir entry FG, zero surface tractions and \( v_x = 0 \); the assumption of zero surface tractions is considered as a good approximation due to the large flat surface area of the slurry in the reservoir; (7) along the reservoir left wall GA, no slip velocity, i.e., \( v_x = v_y = 0 \).

Note that at point A there is a singularity because mathematically the same node has a zero velocity if it belongs to wall FA and a finite \( V_B \) velocity if it belongs to the belt AB. Practically there is a very small gap between the reservoir wall and the belt to allow for belt motion. Numerically, we set very small elements near the corners and set the corner node (point A) as a wall node (zero velocity), and the next node on the belt we set equal to the belt speed. This has been found to be the best way to handle the singularity, which otherwise may affect grossly the results (loss of mass flow rate).

(4) Method of Solution

The solution of the above set of equations along with the boundary conditions has been carried out using the MACVIP finite element program developed originally for viscoelastic flows and modified to account for viscoplastic constitutive equations. MACVIP implements a finite element formulation for the mass and momentum equations using the primitive variable approach, i.e., velocities and pressure \((u-v-p)\) formulation. Streamlines are obtained \emph{a posteriori} by solving the Poisson equation. More details about the solution process can be found in Ref. 17.

IV. Results and Discussion

(1) Vertical Reservoir

The numerical simulations have been initially carried out in the experimental design apparatus of Fig. 2, comprising the slurry reservoir with vertical walls, the channel and the exit region in an effort to compare the results with the experimental findings. The domain exit has been set at 6 mm from the channel exit (or 4H), which was found sufficient to achieve a flat velocity profile with no further changes occurring.

The calculations have been performed with several grids to establish results independent of mesh influence. Typical grids in the entry and exit regions are shown in Fig. 5. They contain over 1000 elements and nearly 4000 unknown degrees of freedom for the velocities—pressures. Many elements are concentrated near the walls and the singularities (points A, D, and E), where most of the drastic changes occur.

The cases studied for three different belt speeds are shown in Table I, along with the relevant dimensionless numbers in the system. It is seen that \( \text{Re} << 1 \), justifying the creeping flow approximation. However, the other dimensionless numbers are intermediate between their extreme values, and their impact on the flow field is important.

In graphical form we begin in Fig. 6 with the depiction of streamline patterns and yielded/unyielded regions in the domain as the belt speed \( V_B \) (or equivalently the flow rate)
increase. The stream function $\psi$ has been made dimensionless between the values of 0 and 1 according to

$$\psi^* = \frac{\psi - \psi_b}{\psi_{\text{max}} - \psi_b} \quad (15)$$

where $\psi_b$ and $\psi_{\text{max}}$ are the stream function values at the support belt and coating surface, respectively. Thus, a value of 1 corresponds to the flow rate and negative values correspond to a percentage of the fluid recirculating in the opposite direction from the main flow. Streamlines are drawn with increments of 0.1 in between.

It is seen that large vortices (recirculatory regions) are always present in the reservoir caused by the dragging action of the belt and the absence of externally imposed pressure forces. In order to quantify the vortex intensity, we can compute the maximum difference between the stream function value in the vortex region (eye of the vortex) and the value at the wall. The relative vortex intensity is then calculated as the ratio of this maximum difference to the flow rate in the main stream according to

$$-\psi_{\text{max}}^* = \frac{\psi_{\text{max}} - \psi_e}{\psi_b - \psi_e} \quad (16)$$

It is seen that the vortex intensity increases drastically with belt speed from 22% to 55% to 86% of the flow rate for the three cases, but there is a modest increase in size as seen in Fig. 6.

The reverse happens to the unyielded regions which decrease with increasing speed, as expected, due to the higher stress levels. The unyielded regions tend to form near the reservoir top, near the right wall, and the upper half of the vortex. At the highest speed of 15.16 mm/s, they have virtually disappeared in the vortex region. The corresponding yielded/unyielded regions after the exit are shown in Fig. 7 along with the streamlines. Increasing the belt speed pushes the unyielded regions farther away from the exit, which is consistent with previous results by Abdali et al., who showed that this is the case as the Bi number decreases.

The pressure drops along the belt are shown in Fig. 8 for the three different belt speeds. Higher speeds produce higher maxima and minima. Note in the beginning the low peaks right at the impact point A of the fluid with the belt, where a singularity exists due to the sudden change in speed from 0 to $V_b$. The pressure increases to the high peaks at the entry point E to the channel, after which it drops monotonically until the exit point D. Most of the pressure drop is due to gravity ($pgh_{\text{inlet}} = 0.43$ Pa). The extra pressure drop is caused by the viscous forces, which are responsible for recirculation. It is also seen that the pressure passes through zero at some distance upstream from the exit to the atmospheric for all belt speeds. This is due to the drag-dominated flow in the channel, which is long enough ($L = 107$ mm) so that all the pressure head generated in the reservoir due to gravity and viscous forces has been used up. Shorter channels would produce coatings determined not solely by the dragging action of the belt but by the pressure forces as well. The final pressure value on the belt after the exit corresponds to $pgh$ due to gravity.

The shear stress distributions along the belt are shown in Fig. 9 for the three different belt speeds. Again negative high peaks appear at the impact region (point A) and then there is a progressive buildup of stresses before entrance to the blade channel (point E) where a leveling-off occurs due to the straight channel geometry. At exit from the blade channel (point D), the shear stresses quickly drop to zero. It is seen that everywhere on the belt the absolute shear stress values exceed the yield stress ($\tau_y = 0.0121$ kPa), except in the free surface region.

A typical comparison between the experimentally found free surface shape and the numerical simulations for $V_b = 7.58$ mm/s (case II) is shown in Fig. 10. A good agreement is obtained, especially for the final coating thickness (error <1%), but the numerical results tend to underestimate the experimental free surface curvature. The discrepancy we believe is mainly due to the use of linear interpolation functions for the surface tension terms in the finite element program. Quadratic interpolation would certainly improve the curvature as $x \to 0$. The other two cases gave similar shapes both experimentally and numerically and are not repeated here.

(2) Tapered Reservoir

It is well known that tapered reservoirs are sometimes used in tape casting to improve the product quality. It is argued here that such geometries are necessary to eliminate the vortices and produce a smooth streamlined flow pattern. Fluid recirculating in large areas may eventually come out of the machine having

![Fig. 6. Streamline patterns in entry flow of ceramic slurry from the reservoir into the blade channel for different belt speeds (unyielded regions are shaded). Data given in Table I.](image-url)
Fig. 7. Streamline patterns in exit flow of ceramic slurry for different belt speeds (unyielded regions are shaded). Data given in Table I.

Fig. 8. Pressure drops along the support belt for different belt speeds in tape casting of ceramic slurry.

Fig. 9. Shear stress distributions along the support belt for different belt speeds in tape casting of ceramic slurry.

The following criteria are thus suggested in order to achieve good properties, independent of location or process time, for the ceramic tapes produced: (1) no plug flow inside the channel; (2) equal shear rates corresponding to drag flow inside the channel; (3) no recirculation in the whole flow field to avoid particle segregation; (4) no dead regions in the whole flow field; (5) almost plug flow in the reservoir to avoid imbalances in the forming process; (6) approximately equal length of all streamlines in the reservoir to achieve equal travel times for the fluid.

Also minor changes in the height of the reservoir should not affect the tape thickness over a certain range of operations.

It is possible to modify the design in order to eliminate recirculation in the reservoir and plug regions in the channel by using the lubrication approximation theory (LAT). LAT reduces the flow domain to one dimension assuming locally a fully developed velocity profile for the material in question. For combined pressure and drag flow of a Bingham plastic in a channel, LAT gives the case of no plug for which

\[ \frac{v_r}{V_{B}} = \frac{\text{Ha}}{2} \left( \frac{y}{H} \right) - 1 \left( \frac{y}{H} - 1 \right) \quad \text{for } \text{Ha} < 2 \]  

(17)

Note also the dependence of Ha on the flow rate Q:

\[ \text{Ha} = 12 \left( \frac{Q}{V_{B}H} - \frac{1}{2} \right) \]  

(18)

Values of \( \text{Ha} < 2 \) can be reached by changing the length L of the channel, so that the pressure gradient \( \Delta P/L \) is small enough.
Fig. 12. Streamline patterns in entry flow of ceramic slurry from the modified tapered reservoir into the blade tapered channel for different belt speeds (unyielded regions are shaded). Design A based on LAT to eliminate recirculation. Design B to eliminate recirculation and reduce unyielded regions. Data given in Table I for the three cases.

to give \( H_a < 2 \). This is necessary when \( H, \mu, \) and \( V_b \) are fixed. The present length of \( L = 107 \) mm is long enough to always give \( H_a < 2 \).

Knowing the rheological constants and support belt speed, it is possible to solve for the maximum height \( H \) of the channel, by requiring that the shear rate become zero on the blade wall. This guarantees that no negative velocities are obtained anywhere in the channel, thus avoiding recirculation. This method applied to the present data and for \( V_b = 7.58 \) mm/s gives \( H_{max} = 23 \) mm.

We have therefore modified the design of Fig. 5 to add a tapered section in the reservoir running parallel to a tapered section of the blade going from \( H \) to \( H_{max} \). Thus, we do not allow the fluid to flow under drag in a channel having a height greater than \( H_{max} \). We have kept about the same reservoir length and the same height. The new design (called design A) and its finite element discretization are shown in Fig. 11. About the same number of elements and nodes have been used as before to ensure results free from mesh influence.

The streamline patterns together with yielded/unyielded regions for different speeds are shown in Fig. 12. The vortices have been successfully suppressed, and a smooth streamline pattern exists for all belt speeds. The unyielded regions are more evenly distributed in the reservoir and are reduced in size as the belt speed increases. At the highest speed, the unyielded regions are limited to the top and near the left reservoir wall.

The corresponding pressure drop along the lower tapered reservoir wall and support belt is shown in Fig. 13 for case II (belt speed \( V_b = 7.58 \) mm/s). Again high peaks appear at impact point with the belt (point I). The channel is long enough to have used up all pressure drop in it, so that the coating thickness is mainly determined by the dragging action of the belt.

A further design modification is possible to eliminate the large unyielded regions on the left reservoir wall as well as the
dead region near the tapered blade channel after entrance. We refer to the new modified design as design B. The corresponding streamline patterns with yielded/unyielded regions are also shown for comparison in Fig. 12 for the three different belt speeds. It is evident that design B satisfies the criteria set forth above, i.e., elimination of recirculation and reduction of the unyielded regions. The pressure drops and shear stress distributions show similar trends as in the previous designs and will not be repeated here.

V. Conclusions

The flow of ceramic slurries inside tape casting equipment has been analyzed using the finite element method. Rheological data for an organic-bonded alumina slurry [α-Al₂O₃ powder, cellulose acetate (binder), dibutyl phthalate (plasticizer), fish oil, acetone (solvent), and cyclohexanone (solvent)] have been fitted with a viscoplastic Bingham model incorporating a stress growth exponent which makes it valid in both yielded and unyielded regions. The full 2-D analysis gives a good agreement between the computed free surface profiles and the experimental observations. The extent and shape of yielded/unyielded regions is also found for different support belt speeds. The computations show that reservoirs with vertical walls and an open side to the moving belt have always large vortices in the absence of externally imposed pressure forces (drag-dominated flow caused by the belt). Elimination of these vortices is possible by modifications in the reservoir and blade design, incorporating tapered walls to produce smooth streamline patterns. The use of the lubrication approximation theory (LAT) is useful for easily determining design changes to meet certain criteria. As such we set forth here mainly the absence of recirculation in the reservoir and the reduction of unyielded regions in order to produce ceramic tapes free of defects caused by the rheological flow behavior of the slurries.

APPENDIX

The lubrication approximation theory (LAT) gives for the case of a fully developed combined pressure and drag flow of a viscoplastic Bingham fluid in a straight channel\(^9\) (see also Fig. A1)

\[
\text{const.} = \frac{dP}{dx} = \frac{d\tau_y}{dy} \quad (A-1)
\]

where

\[
\tau_y = \pm \tau_r + \mu \dot{\gamma}, \quad \text{(for} \ \tau_y > \tau_r \text{)} \quad (A-2a)
\]

\[
\frac{d\tau_y}{dy} = \dot{\gamma} = 0 \quad \text{(for} \ \tau_y \leq \tau_r \text{)} \quad (A-2b)
\]

and the boundary conditions are

\[
(BC1) \ \ \tau_y(y) = V_b \quad (at \ y = 0) \quad (A-3a)
\]

\[
(BC2) \ \ \tau_y(y) = 0 \quad (at \ y = H) \quad (A-3b)
\]

By setting dimensionless variables

\[
x^* = \frac{x}{L}, \ \ \ y^* = \frac{y}{H}, \ \ \ \tau^* = \frac{\tau_y}{\tau_r}, \ \ \ \nu^* = \frac{\nu}{V_b} \quad (A-4)
\]

\[
\frac{\tau_rH}{\mu V_b^2} \quad \text{Ha} = \frac{(\Delta P/L)H^2}{\mu V_b^2}, \ \ HB = \frac{Ha}{Bi} = \frac{(\Delta P/L)H}{\tau_r} \quad (A-5)
\]

and dropping thereafter the asterisks we obtain upon integration of Eq. (A-1)

\[
\tau = HB(y_o - y) \quad (A-6)
\]

where \(y_o\) is the position of zero shear stress.

From Eqs. (A-2a) and (A-6) we obtain

\[
\pm 1 + \frac{1}{Bi} \frac{dy}{dy} = HB(y_o - y) \quad (A-7)
\]

and the location of the plug region \(y_p\) when \(dy/dy = 0\), i.e.

\[
y_{p1} = y_o - 1/\HB \quad (A-8a)
\]

\[
y_{p2} = y_o + 1/\HB \quad (A-8b)
\]

For the solution of Eq. (A-7) we can distinguish the following three cases:

Case A—no plug, \(Ha < 2\)

\[
y_o = \frac{1 - Bi}{2} + \frac{1}{2} \quad (A-9)
\]

\[
\nu(y) = \left(\frac{Ha}{2y - 1}\right)(y - 1) \quad (A-10)
\]

Case B—plug on the wall, \(2 < Ha < Ha_{o-c}\) where

\[
Ha_{o-c} = 2Bi + 1 + \sqrt{4Bi + 1} \quad (A-11)
\]

\[
y_o = -1 - \frac{1}{HB} - \frac{1}{\sqrt{Ha}} \quad (A-12)
\]
\[ v(y) = \frac{H_a}{2} \left( 1 - y - 2 \sqrt{\frac{2}{H_a}} (y - 1) \right) \] (A-13a)

\[ (y_{p2} \leq y \leq 1) \]

\[ v(y) = 1 \] (A-13b)

\[ (0 \leq y \leq y_{p1}) \]

Case C—plug in the middle of the flow, \( H_a > H_{a_0} - c \)

\[ y_0 = \frac{H_a}{2 Bi - H_a} \frac{1}{2} \] (A-14)

\[ v(y) = \frac{H_a}{2} y \left( 2 \left( y_0 - \frac{Bi}{H_a} \right) - y \right) + 1 \] (A-15a)

\[ (0 \leq y \leq y_{p1}) \]

\[ v(y) = \frac{H_a}{8} + \frac{Bi}{2} \frac{1}{2H_a} + \frac{H_a}{Ha - 2Bi} \]

\[ \times \left[ \frac{1}{2} - \frac{1}{ HB} + \frac{1}{2(2(Ha - 2Bi))} \right] \] (A-15b)

\[ (y_{p1} \leq y \leq y_{p2}) \]

\[ v(y) = \left( H_{a0} - \frac{H_a}{2} (y + 1) + Bi \right) (y - 1) \] (A-15c)

\[ (y_{p2} \leq y \leq 1) \]

References